OCR MEI Maths FP1

Past Paper Pack

2005-2014

OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4755

Further Concepts For Advanced Mathematics (FP1)

Friday **21 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Registered Charity 1066969
[Turn over

Section A (36 marks)

1 You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ -2 & 1 \end{pmatrix}$.

Find the inverse of **M**.

The transformation associated with **M** is applied to a figure of area 2 square units. What is the area of the transformed figure? [3]

2 **(i)** Show that $\frac{1}{r+1} - \frac{1}{r+2} = \frac{1}{(r+1)(r+2)}$. [2]

(ii) Hence use the method of differences to find the sum of the series

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)}.$$ [4]

3 **(i)** Solve the equation $\frac{1}{x+2} = 3x + 4$. [3]

(ii) Solve the inequality $\frac{1}{x+2} \leqslant 3x + 4$. [4]

4 Find $\sum_{r=1}^{n} r^2(r+2)$, giving your answer in a factorised form. [6]

5 The roots of the cubic equation $x^3 + 2x^2 + x - 3 = 0$ are α, β and γ.

Find the cubic equation whose roots are $\alpha + 1, \beta + 1$ and $\gamma + 1$, simplifying your answer as far as you can. [6]

6 Prove by induction that $\sum_{r=1}^{n} r2^{r-1} = 1 + (n-1)2^n$. [8]

Section B (36 marks)

7 A curve has equation $y = \dfrac{(2x-3)(x+1)}{(x+4)(x-2)}$.

(i) Write down the values of x for which $y = 0$. [1]

(ii) Write down the equations of the three asymptotes. [3]

(iii) Determine whether the curve approaches the horizontal asymptote from above or from below for

(*A*) large positive values of x,

(*B*) large negative values of x. [3]

(iv) Sketch the curve. [3]

(v) Solve the inequality $\dfrac{(2x-3)(x+1)}{(x+4)(x-2)} \leqslant 2$. [4]

8 Two complex numbers are given by $\alpha = 2 - \mathrm{j}$ and $\beta = -1 + 2\mathrm{j}$.

(i) Find $\alpha + \beta$, $\alpha\beta$ and $\dfrac{\alpha}{\beta}$ in the form $a + b\mathrm{j}$, showing your working. [6]

(ii) Find the modulus of α, leaving your answer in surd form. Find also the argument of α. [2]

(iii) Sketch the locus $|z - \alpha| = 2$ on an Argand diagram. [2]

(iv) On a separate Argand diagram, sketch the locus $\arg(z - \beta) = \frac{1}{4}\pi$. [2]

[Turn over

9 You are given the matrix $\mathbf{M} = \begin{pmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix}$.

(i) Calculate $\mathbf{M}^2$. [1]

You are now given that the matrix **M** represents a reflection in a line through the origin.

(ii) Explain how your answer to part **(i)** relates to this information. [1]

(iii) By investigating the invariant points of the reflection, find the equation of the mirror line. [3]

(iv) Describe fully the transformation represented by the matrix $\mathbf{P} = \begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$. [2]

(v) A composite transformation is formed by the transformation represented by **P** followed by the transformation represented by **M**. Find the single matrix that represents this composite transformation. [2]

(vi) The composite transformation described in part **(v)** is equivalent to a single reflection. What is the equation of the mirror line of this reflection? [1]

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Further Concepts For Advanced Mathematics (FP1)

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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Registered Charity 1066969
[Turn over

Section A (36 marks)

1 (i) Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$. [2]

(ii) Use this inverse to solve the simultaneous equations

$$4x + 3y = 5,$$
$$x + 2y = -4,$$

showing your working clearly. [3]

2 Find the roots of the quadratic equation $x^2 - 8x + 17 = 0$ in the form $a + b\mathrm{j}$.

Express these roots in modulus-argument form. [5]

3 Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$. [3]

4 The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]

(ii) Hence find the value of $\alpha^2 + \beta^2$. [2]

(iii) Find a quadratic equation which has roots 2α and 2β. [2]

5 (i) Sketch the locus $|z - (3 + 4\mathrm{j})| = 2$ on an Argand diagram. [2]

(ii) On the same diagram, sketch the locus $\arg(z - 4) = \frac{1}{2}\pi$. [2]

(iii) Indicate clearly on your sketch the points which satisfy **both**

$$|z - (3 + 4\mathrm{j})| = 2 \quad \textbf{and} \quad \arg(z - 4) = \tfrac{1}{2}\pi.$$ [1]

6 Prove by induction that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$. [7]

7 Find $\sum_{r=1}^{n} 3r(r-1)$, expressing your answer in a fully factorised form. [6]

Section B (36 marks)

8 A curve has equation $y = \dfrac{x^2 - 4}{(3x - 2)^2}$.

(i) Find the equations of the asymptotes. [2]

(ii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your description. [3]

(iii) Sketch the curve. [5]

(iv) Solve the inequality $\dfrac{x^2 - 4}{(3x - 2)^2} \geqslant -1$. [4]

9 The quartic equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$, where A, B, C and D are real numbers, has roots $2 + \mathrm{j}$ and $-2\mathrm{j}$.

(i) Write down the other roots of the equation. [2]

(ii) Find the values of A, B, C and D. [8]

10 **(i)** You are given that

$$\frac{2}{r(r+1)(r+2)} = \frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}.$$

Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}.$$ [9]

(ii) Hence find the sum of the infinite series

$$\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \ldots.$$ [3]

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MEI STRUCTURED MATHEMATICS 4755

Further Concepts For Advanced Mathematics (FP1)

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
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Registered Charity 1066969
[Turn over

Section A (36 marks)

1 You are given that $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Calculate, where possible, $2\mathbf{B}$, $\mathbf{A} + \mathbf{C}$, $\mathbf{CA}$ and $\mathbf{A} - \mathbf{B}$. [5]

(ii) Show that matrix multiplication is not commutative. [2]

2 **(i)** Given that $z = a + b\mathrm{j}$, express $|z|$ and z^* in terms of a and b. [2]

(ii) Prove that $zz^* - |z|^2 = 0$. [3]

3 Find $\sum_{r=1}^{n} (r+1)(r-1)$, expressing your answer in a fully factorised form. [6]

4 The matrix equation $\begin{pmatrix} 6 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ represents two simultaneous linear equations in x and y.

(i) Write down the two equations. [2]

(ii) Evaluate the determinant of $\begin{pmatrix} 6 & -2 \\ -3 & 1 \end{pmatrix}$.

What does this value tell you about the solution of the equations in part **(i)**? [3]

5 The cubic equation $x^3 + 3x^2 - 7x + 1 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [2]

(ii) Find the cubic equation with roots 2α, 2β and 2γ, simplifying your answer as far as possible. [4]

6 Prove by induction that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. [7]

Section B (36 marks)

7 A curve has equation $y = \dfrac{3 + x^2}{4 - x^2}$.

(i) Show that y can never be zero. [1]

(ii) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. [3]

(iii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your description. [2]

(iv) Sketch the curve. [3]

(v) Solve the inequality $\dfrac{3 + x^2}{4 - x^2} \leqslant -2$. [4]

8 You are given that the complex number $\alpha = 1 + \mathrm{j}$ satisfies the equation $z^3 + 3z^2 + pz + q = 0$, where p and q are real constants.

(i) Find α^2 and α^3 in the form $a + b\mathrm{j}$. Hence show that $p = -8$ and $q = 10$. [6]

(ii) Find the other two roots of the equation. [3]

(iii) Represent the three roots on an Argand diagram. [2]

[Turn over

9 A transformation T acts on all points in the plane. The image of a general point P is denoted by P′.

P′ always lies on the line $y = 2x$ and has the same y-coordinate as P. This is illustrated in Fig. 9.

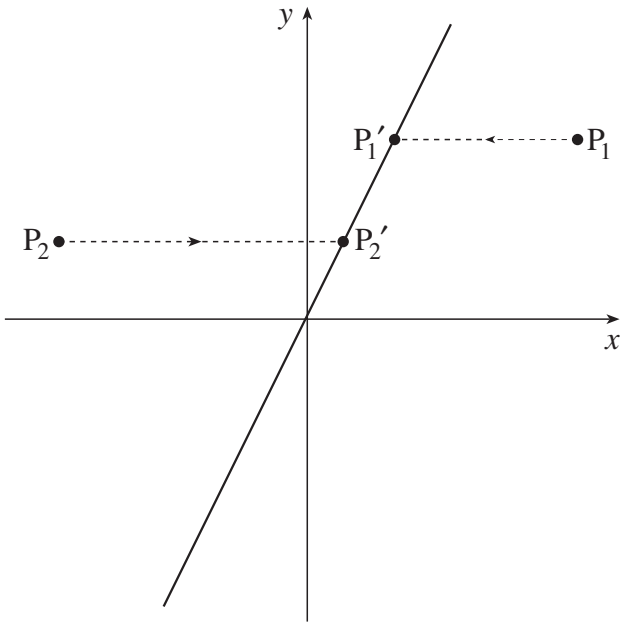

Fig. 9

(i) Write down the image of the point $(10, 50)$ under transformation T. [1]

(ii) P has coordinates (x, y). State the coordinates of P′. [2]

(iii) All points on a particular line l are mapped onto the point $(3, 6)$. Write down the equation of the line l. [1]

(iv) In part **(iii)**, the whole of the line l was mapped by T onto a single point. There are an infinite number of lines which have this property under T. Describe these lines. [1]

(v) For a different set of lines, the transformation T has the same effect as translation parallel to the x-axis. Describe this set of lines. [1]

(vi) Find the 2×2 matrix which represents the transformation. [3]

(vii) Show that this matrix is singular. Relate this result to the transformation. [3]

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MEI STRUCTURED MATHEMATICS 4755

Further Concepts for Advanced Mathematics (FP1)

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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Registered Charity 1066969
[Turn over

Section A (36 marks)

1 **(i)** State the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. [1]

(ii) Write down the 2×2 matrix for rotation through 90° anticlockwise about the origin. [1]

(iii) Find the 2×2 matrix for rotation through 90° anticlockwise about the origin, followed by reflection in the x-axis. [2]

2 Find the values of A, B, C and D in the identity

$$2x^3 - 3x^2 + x - 2 \equiv (x + 2)(Ax^2 + Bx + C) + D.$$ [5]

3 The cubic equation $z^3 + 4z^2 - 3z + 1 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

(ii) Show that $\alpha^2 + \beta^2 + \gamma^2 = 22$. [3]

4 Indicate, on separate Argand diagrams,

(i) the set of points z for which $|z-(3-\mathrm{j})| \leqslant 3$, [3]

(ii) the set of points z for which $1 < |z-(3-\mathrm{j})| \leqslant 3$, [2]

(iii) the set of points z for which $\arg(z-(3-\mathrm{j})) = \frac{1}{4}\pi$. [3]

5 **(i)** The matrix $\mathbf{S} = \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$ represents a transformation.

(*A*) Show that the point $(1, 1)$ is invariant under this transformation. [1]

(*B*) Calculate $\mathbf{S}^{-1}$. [2]

(*C*) Verify that $(1, 1)$ is also invariant under the transformation represented by $\mathbf{S}^{-1}$. [1]

(ii) Part **(i)** may be generalised as follows.

If (x, y) is an invariant point under a transformation represented by the non-singular matrix $\mathbf{T}$, it is also invariant under the transformation represented by $\mathbf{T}^{-1}$.

Starting with $\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$, or otherwise, prove this result. [2]

6 Prove by induction that $3 + 6 + 12 + \ldots + 3 \times 2^{n-1} = 3(2^n - 1)$ for all positive integers n. [7]

Section B (36 marks)

7 A curve has equation $y = \dfrac{x^2}{(x-2)(x+1)}$.

(i) Write down the equations of the three asymptotes. [3]

(ii) Determine whether the curve approaches the horizontal asymptote from above or from below for

(*A*) large positive values of x,

(*B*) large negative values of x. [3]

(iii) Sketch the curve. [4]

(iv) Solve the inequality $\dfrac{x^2}{(x-2)(x+1)} > 0$. [3]

8 **(i)** Verify that $2 + \mathrm{j}$ is a root of the equation $2x^3 - 11x^2 + 22x - 15 = 0$. [5]

(ii) Write down the other complex root. [1]

(iii) Find the third root of the equation. [4]

9 **(i)** Show that $r(r+1)(r+2) - (r-1)r(r+1) \equiv 3r(r+1)$. [2]

(ii) Hence use the method of differences to find an expression for $\sum_{r=1}^{n} r(r+1)$. [6]

(iii) Show that you can obtain the same expression for $\sum_{r=1}^{n} r(r+1)$ using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$. [5]

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RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

THURSDAY 18 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

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[Turn over

Section A (36 marks)

1 Is the following statement true or false? Justify your answer.

$$x^2 = 4 \text{ if and only if } x = 2$$ [2]

2 **(i)** Find the roots of the quadratic equation $z^2 - 4z + 7 = 0$, simplifying your answers as far as possible. [4]

(ii) Represent these roots on an Argand diagram. [2]

3 The points A, B and C in the triangle in Fig. 3 are mapped to the points A′, B′ and C′ respectively under the transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$.

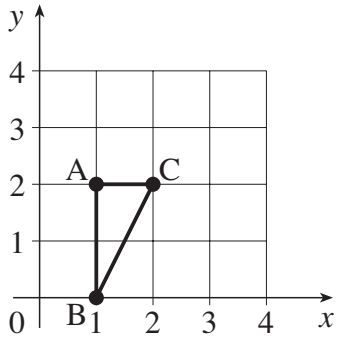

Fig. 3

(i) Draw a diagram showing the image of the triangle after the transformation, labelling the image of each point clearly. [4]

(ii) Describe fully the transformation represented by the matrix **M**. [3]

4 Use standard series formulae to find $\sum_{r=1}^{n} r(r^2 + 1)$, factorising your answer as far as possible. [6]

5 The roots of the cubic equation $2x^3 - 3x^2 + x - 4 = 0$ are α, β and γ.

Find the cubic equation whose roots are $2\alpha + 1$, $2\beta + 1$ and $2\gamma + 1$, expressing your answer in a form with integer coefficients. [7]

6 Prove by induction that $\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$. [8]

Section B (36 marks)

7 A curve has equation $y = \dfrac{5}{(x+2)(4-x)}$.

(i) Write down the value of y when $x = 0$. [1]

(ii) Write down the equations of the three asymptotes. [3]

(iii) Sketch the curve. [3]

(iv) Find the values of x for which $\dfrac{5}{(x+2)(4-x)} = 1$ and hence solve the inequality

$$\frac{5}{(x+2)(4-x)} < 1.$$ [5]

8 It is given that $m = -4 + 2\mathrm{j}$.

(i) Express $\dfrac{1}{m}$ in the form $a + b\mathrm{j}$. [2]

(ii) Express m in modulus-argument form. [4]

(iii) Represent the following loci on separate Argand diagrams.

(*A*) $\arg(z - m) = \dfrac{\pi}{4}$ [2]

(*B*) $0 < \arg(z - m) < \dfrac{\pi}{4}$ [3]

9 Matrices **M** and **N** are given by $\mathbf{M} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix}$.

(i) Find $\mathbf{M}^{-1}$ and $\mathbf{N}^{-1}$. [3]

(ii) Find **MN** and $(\mathbf{MN})^{-1}$. Verify that $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$. [6]

(iii) The result $(\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ is true for any two 2×2, non-singular matrices **P** and **Q**.

The first two lines of a proof of this general result are given below. Beginning with these two lines, complete the general proof.

$$(\mathbf{PQ})^{-1}\mathbf{PQ} = \mathbf{I}$$
$$\Rightarrow \quad (\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1} = \mathbf{IQ}^{-1}$$ [4]

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RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

MONDAY 11 JUNE 2007 Afternoon

Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

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 [Turn over

Section A (36 marks)

1 You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$.

(i) Find the inverse of **M**. [2]

(ii) A triangle of area 2 square units undergoes the transformation represented by the matrix **M**. Find the area of the image of the triangle following this transformation. [1]

2 Write down the equation of the locus represented by the circle in the Argand diagram shown in Fig. 2. [3]

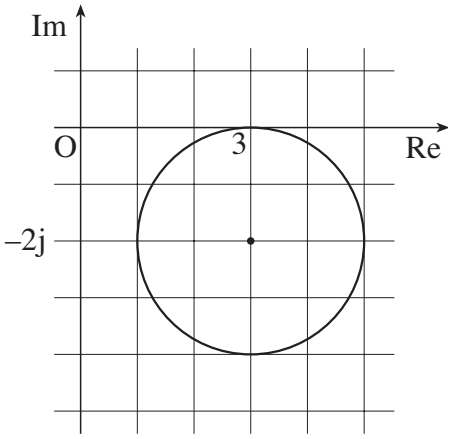

Fig. 2

3 Find the values of the constants A, B, C and D in the identity

$$x^3 - 4 \equiv (x - 1)(Ax^2 + Bx + C) + D.$$ [5]

4 Two complex numbers, α and β, are given by $\alpha = 1 - 2\mathrm{j}$ and $\beta = -2 - \mathrm{j}$.

(i) Represent β and its complex conjugate β^* on an Argand diagram. [2]

(ii) Express $\alpha\beta$ in the form $a + b\mathrm{j}$. [2]

(iii) Express $\dfrac{\alpha + \beta}{\beta}$ in the form $a + b\mathrm{j}$. [3]

5 The roots of the cubic equation $x^3 + 3x^2 - 7x + 1 = 0$ are α, β and γ. Find the cubic equation whose roots are 3α, 3β and 3γ, expressing your answer in a form with integer coefficients. [6]

6 **(i)** Show that $\frac{1}{r+2} - \frac{1}{r+3} = \frac{1}{(r+2)(r+3)}$. [2]

(ii) Hence use the method of differences to find $\frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \ldots + \frac{1}{52 \times 53}$. [4]

7 Prove by induction that $\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$. [6]

Section B (36 marks)

8 A curve has equation $y = \frac{x^2 - 4}{(x-3)(x+1)(x-1)}$.

(i) Write down the coordinates of the points where the curve crosses the axes. [3]

(ii) Write down the equations of the three vertical asymptotes and the one horizontal asymptote. [4]

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(*A*) large positive values of x,

(*B*) large negative values of x. [3]

(iv) Sketch the curve. [4]

9 The cubic equation $x^3 + Ax^2 + Bx + 15 = 0$, where A and B are real numbers, has a root $x = 1 + 2\mathrm{j}$.

(i) Write down the other complex root. [1]

(ii) Explain why the equation must have a real root. [1]

(iii) Find the value of the real root and the values of A and B. [9]

[Question 10 is printed overleaf.]

[Turn over

10 You are given that $\mathbf{A} = \begin{pmatrix} 1 & -2 & k \\ 2 & 1 & 2 \\ 3 & 2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -5 & -2+2k & -4-k \\ 8 & -1-3k & -2+2k \\ 1 & -8 & 5 \end{pmatrix}$ and that $\mathbf{AB}$ is of the form $\mathbf{AB} = \begin{pmatrix} k-n & 0 & 0 \\ 0 & k-n & 0 \\ 0 & 0 & k-n \end{pmatrix}$.

(i) Find the value of n. [2]

(ii) Write down the inverse matrix $\mathbf{A}^{-1}$ and state the condition on k for this inverse to exist. [4]

(iii) Using the result from part **(ii)**, or otherwise, solve the following simultaneous equations.

$$\begin{aligned} x - 2y + z &= 1 \\ 2x + y + 2z &= 12 \\ 3x + 2y - z &= 3 \end{aligned}$$

[5]

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RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
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MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 You are given that matrix $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ and matrix $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$.

(i) Find **BA**. [2]

(ii) A plane shape of area 3 square units is transformed using matrix **A**. The image is transformed using matrix **B**. What is the area of the resulting shape? [3]

2 You are given that $\alpha = -3 + 4\text{j}$.

(i) Calculate α^2. [2]

(ii) Express α in modulus-argument form. [3]

3 **(i)** Show that $z = 3$ is a root of the cubic equation $z^3 + z^2 - 7z - 15 = 0$ and find the other roots. [5]

(ii) Show the roots on an Argand diagram. [2]

4 Using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{n}[(r+1)(r-2)] = \frac{1}{3}n(n^2 - 7)$. [6]

5 The equation $x^3 + px^2 + qx + r = 0$ has roots α, β and γ, where

$$\alpha + \beta + \gamma = 3,$$
$$\alpha\beta\gamma = -7,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 13.$$

(i) Write down the values of p and r. [2]

(ii) Find the value of q. [3]

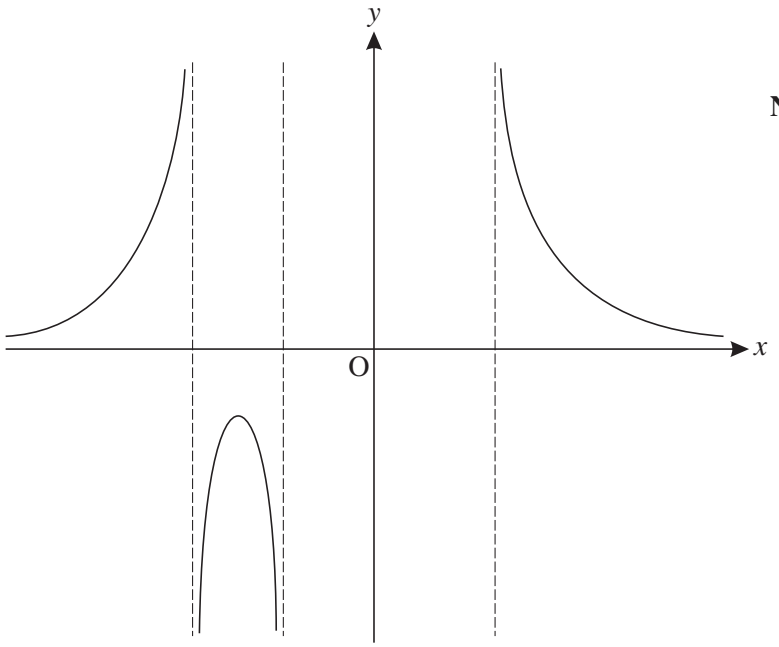
6 A sequence is defined by $a_1 = 7$ and $a_{k+1} = 7a_k - 3$.

(i) Calculate the value of the third term, a_3. [2]

(ii) Prove by induction that $a_n = \dfrac{(13 \times 7^{n-1}) + 1}{2}$. [6]

Section B (36 marks)

7 The sketch below shows part of the graph of $y = \dfrac{x-1}{(x-2)(x+3)(2x+3)}$. One section of the graph has been omitted.



Not to scale

Fig. 7

(i) Find the coordinates of the points where the curve crosses the axes. [2]

(ii) Write down the equations of the three vertical asymptotes and the one horizontal asymptote. [4]

(iii) Copy the sketch and draw in the missing section. [2]

(iv) Solve the inequality $\dfrac{x-1}{(x-2)(x+3)(2x+3)} \geqslant 0$. [3]

8 **(i)** On a single Argand diagram, sketch the locus of points for which

(*A*) $|z - 3\mathrm{j}| = 2$, [3]

(*B*) $\arg(z+1) = \frac{1}{4}\pi$. [3]

(ii) Indicate clearly on your Argand diagram the set of points for which

$$|z - 3\mathrm{j}| \leqslant 2 \quad \text{and} \quad \arg(z+1) \leqslant \tfrac{1}{4}\pi.$$ [2]

(iii) (*A*) By drawing an appropriate line through the origin, indicate on your Argand diagram the point for which $|z - 3\mathrm{j}| = 2$ and $\arg z$ has its minimum possible value. [2]

(*B*) Calculate the value of $\arg z$ at this point. [2]

9 A transformation T acts on all points in the plane. The image of a general point P is denoted by P′. P′ always lies on the line $y = x$ and has the same x-coordinate as P. This is illustrated in Fig. 9.

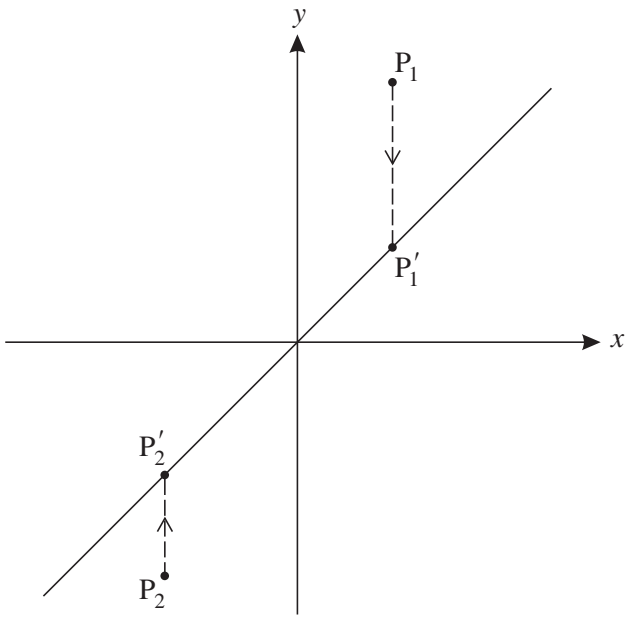


Fig. 9

(i) Write down the image of the point $(-3, 7)$ under transformation T. [1]

(ii) Write down the image of the point (x, y) under transformation T. [2]

(iii) Find the 2×2 matrix which represents the transformation. [3]

(iv) Describe the transformation M represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [2]

(v) Find the matrix representing the composite transformation of T followed by M. [2]

(vi) Find the image of the point (x, y) under this composite transformation. State the equation of the line on which all of these images lie. [3]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
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This document consists of **4** printed pages.

Section A (36 marks)

1 **(i)** Write down the matrix for reflection in the y-axis. [1]

(ii) Write down the matrix for enlargement, scale factor 3, centred on the origin. [1]

(iii) Find the matrix for reflection in the y-axis, followed by enlargement, scale factor 3, centred on the origin. [2]

2 Indicate on a single Argand diagram

(i) the set of points for which $|z - (-3 + 2\mathrm{j})| = 2$, [3]

(ii) the set of points for which $\arg(z - 2\mathrm{j}) = \pi$, [3]

(iii) the two points for which $|z - (-3 + 2\mathrm{j})| = 2$ and $\arg(z - 2\mathrm{j}) = \pi$. [1]

3 Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}$. [3]

4 Find the values of A, B, C and D in the identity $3x^3 - x^2 + 2 \equiv A(x-1)^3 + (x^3 + Bx^2 + Cx + D)$. [5]

5 You are given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 2 & 5 \\ 4 & 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & 0 & 2 \\ 14 & -14 & 7 \\ -5 & 7 & -4 \end{pmatrix}$.

(i) Calculate $\mathbf{AB}$. [3]

(ii) Write down $\mathbf{A}^{-1}$. [2]

6 The roots of the cubic equation $2x^3 + x^2 - 3x + 1 = 0$ are α, β and γ. Find the cubic equation whose roots are 2α, 2β and 2γ, expressing your answer in a form with integer coefficients. [5]

7 **(i)** Show that $\dfrac{1}{3r-1} - \dfrac{1}{3r+2} \equiv \dfrac{3}{(3r-1)(3r+2)}$ for all integers r. [2]

(ii) Hence use the method of differences to find $\displaystyle\sum_{r=1}^{n} \frac{1}{(3r-1)(3r+2)}$. [5]

Section B (36 marks)

8 A curve has equation $y = \dfrac{2x^2}{(x-3)(x+2)}$.

(i) Write down the equations of the three asymptotes. [3]

(ii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. [3]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\dfrac{2x^2}{(x-3)(x+2)} < 0$. [3]

9 Two complex numbers, α and β, are given by $\alpha = 2 - 2\text{j}$ and $\beta = -1 + \text{j}$.

α and β are both roots of a quartic equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$, where A, B, C and D are real numbers.

(i) Write down the other two roots. [2]

(ii) Represent these four roots on an Argand diagram. [2]

(iii) Find the values of A, B, C and D. [7]

10 **(i)** Using the standard formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, prove that

$$\sum_{r=1}^{n} r^2(r+1) = \tfrac{1}{12}n(n+1)(n+2)(3n+1). \quad [5]$$

(ii) Prove the same result by mathematical induction. [8]

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ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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- Use black ink. Pencil may be used for graphs and diagrams only.
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Section A (36 marks)

1 **(i)** Find the roots of the quadratic equation $z^2 - 6z + 10 = 0$ in the form $a + b\text{j}$. **[2]**

(ii) Express these roots in modulus-argument form. **[3]**

2 Find the values of A, B and C in the identity $2x^2 - 13x + 25 \equiv A(x - 3)^2 - B(x - 2) + C$. **[4]**

3 Fig. 3 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

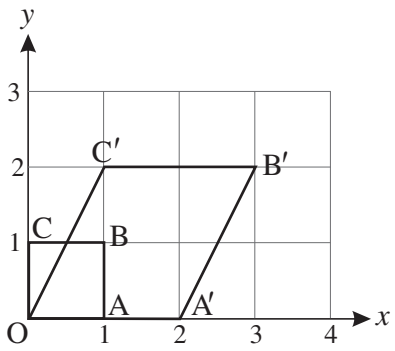

Fig. 3

(i) Write down the matrix **P** representing this transformation. **[1]**

(ii) The parallelogram OA′B′C′ is transformed by the matrix $\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$. Find the coordinates of the vertices of its image, OA″B″C″, following this transformation. **[2]**

(iii) Describe fully the transformation represented by **QP**. **[2]**

4 Write down the equation of the locus represented in the Argand diagram shown in Fig. 4. **[3]**

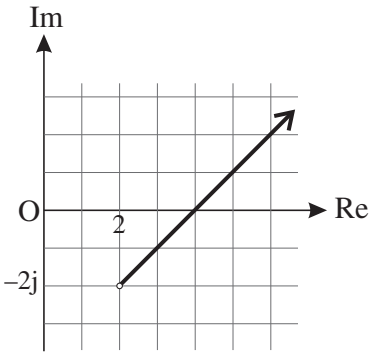

Fig. 4

5 The cubic equation $x^3 - 5x^2 + px + q = 0$ has roots α, -3α and $\alpha + 3$. Find the values of α, p and q. **[6]**

6 Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \tfrac{1}{4}n(n+1)(n+3)(n-2).$$ **[6]**

7 Prove by induction that $12 + 36 + 108 + \ldots + 4 \times 3^n = 6(3^n - 1)$ for all positive integers n. **[7]**

Section B (36 marks)

8 Fig. 8 shows part of the graph of $y = \dfrac{x^2 - 3}{(x-4)(x+2)}$. Two sections of the graph have been omitted.

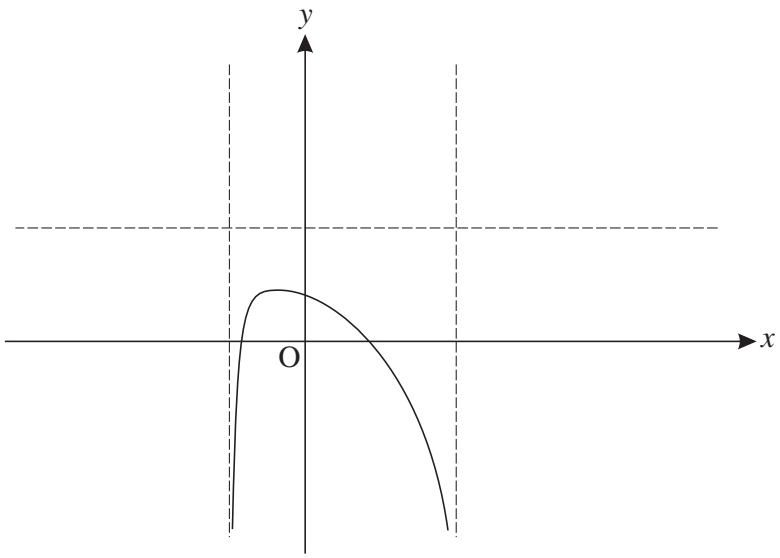

Fig. 8

(i) Write down the coordinates of the points where the curve crosses the axes. **[2]**

(ii) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

(iii) Copy Fig. 8 and draw in the two missing sections. **[4]**

(iv) Solve the inequality $\dfrac{x^2 - 3}{(x-4)(x+2)} \leqslant 0$. **[3]**

[Questions 9 and 10 are printed overleaf.]

9 Two complex numbers, α and β, are given by $\alpha = 1 + \mathrm{j}$ and $\beta = 2 - \mathrm{j}$.

(i) Express $\alpha + \beta$, $\alpha\alpha^*$ and $\dfrac{\alpha + \beta}{\alpha}$ in the form $a + b\mathrm{j}$. **[5]**

(ii) Find a quadratic equation with roots α and α^*. **[2]**

(iii) α and β are roots of a quartic equation with real coefficients. Write down the two other roots and find this quartic equation in the form $z^4 + Az^3 + Bz^2 + Cz + D = 0$. **[5]**

10 You are given that $\mathbf{A} = \begin{pmatrix} 3 & 4 & -1 \\ 1 & -1 & k \\ -2 & 7 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 5+k \\ -5 & 29 & 7 \end{pmatrix}$ and that $\mathbf{AB}$ is of the form $\mathbf{AB} = \begin{pmatrix} 42 & \alpha & 4k-8 \\ 10-5k & -16+29k & -12+6k \\ 0 & 0 & \beta \end{pmatrix}$.

(i) Show that $\alpha = 0$ and $\beta = 28 + 7k$. **[3]**

(ii) Find $\mathbf{AB}$ when $k = 2$. **[2]**

(iii) For the case when $k = 2$ write down the matrix $\mathbf{A}^{-1}$. **[3]**

(iv) Use the result from part **(iii)** to solve the following simultaneous equations. **[4]**

$$\begin{aligned} 3x + 4y - z &= 1 \\ x - y + 2z &= -9 \\ -2x + 7y - 3z &= 26 \end{aligned}$$

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ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 22 May 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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Section A (36 marks)

1 **(i)** Find the inverse of the matrix $\mathbf{M} = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$. **[2]**

(ii) Use this inverse to solve the simultaneous equations

$$4x - y = 49,$$
$$3x + 2y = 100,$$

showing your working clearly. **[3]**

2 Show that $z = 3$ is a root of the cubic equation $z^3 + z^2 - 7z - 15 = 0$ and find the other roots. **[5]**

3 **(i)** Sketch the graph of $y = \dfrac{2}{x+4}$. **[2]**

(ii) Solve the inequality

$$\frac{2}{x+4} \leqslant x + 3,$$

showing your working clearly. **[5]**

4 The roots of the cubic equation $2x^3 + x^2 + px + q = 0$ are $2w$, $-6w$ and $3w$. Find the values of the roots and the values of p and q. **[6]**

5 **(i)** Show that $\dfrac{1}{5r-2} - \dfrac{1}{5r+3} \equiv \dfrac{5}{(5r-2)(5r+3)}$ for all integers r. **[2]**

(ii) Hence use the method of differences to show that $\displaystyle\sum_{r=1}^{n} \frac{1}{(5r-2)(5r+3)} = \frac{n}{3(5n+3)}$. **[4]**

6 Prove by induction that $3 + 10 + 17 + \ldots + (7n - 4) = \frac{1}{2}n(7n - 1)$ for all positive integers n. **[7]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{(x+2)(3x-5)}{(2x+1)(x-1)}$.

(i) Write down the coordinates of the points where the curve crosses the axes. **[3]**

(ii) Write down the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(*A*) large positive values of x,

(*B*) large negative values of x. **[3]**

(iv) Sketch the curve. **[3]**

8 Fig. 8 shows an Argand diagram.

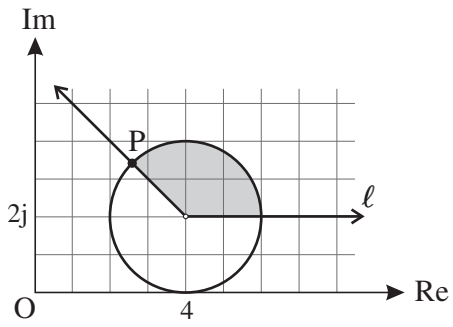

Fig. 8

(i) Write down the equation of the locus represented by the perimeter of the circle in the Argand diagram. **[3]**

(ii) Write down the equation of the locus represented by the half-line ℓ in the Argand diagram. **[3]**

(iii) Express the complex number represented by the point P in the form $a + b\mathrm{j}$, giving the exact values of a and b. **[3]**

(iv) Use inequalities to describe the set of points that fall within the shaded region (excluding its boundaries) in the Argand diagram. **[3]**

[Question 9 is printed overleaf.]

9 You are given that $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$, $\mathbf{N} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(i) The matrix products **Q**(**MN**) and (**QM**)**N** are identical. What property of matrix multiplication does this illustrate?

Find **QMN**. **[4]**

M, **N** and **Q** represent the transformations M, N and Q respectively.

(ii) Describe the transformations M, N and Q. **[4]**

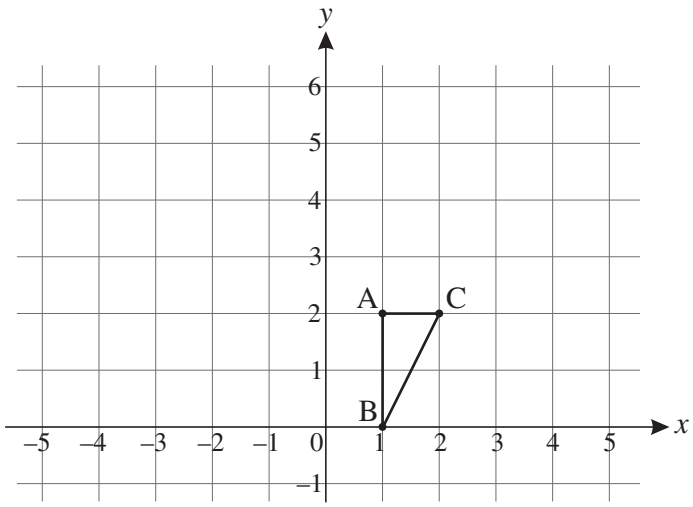

Fig. 9

(iii) The points A, B and C in the triangle in Fig. 9 are mapped to the points A′, B′ and C′ respectively by the composite transformation N followed by M followed by Q. Draw a diagram showing the image of the triangle after this composite transformation, labelling the image of each point clearly. **[4]**

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RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
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Section A (36 marks)

1 Two complex numbers are given by $\alpha = -3 + \mathrm{j}$ and $\beta = 5 - 2\mathrm{j}$.

Find $\alpha\beta$ and $\dfrac{\alpha}{\beta}$, giving your answers in the form $a + b\mathrm{j}$, showing your working. **[5]**

2 You are given that $\mathbf{A} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 5 & 1 & 8 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}$.

(i) Calculate, where they exist, **AB**, **CA**, **B** + **D** and **AC** and indicate any that do not exist. **[5]**

(ii) Matrices **B** and **D** represent transformations B and D respectively. Find the single matrix that represents transformation B followed by transformation D. **[2]**

3 The roots of the cubic equation $4x^3 - 12x^2 + kx - 3 = 0$ may be written $a - d$, a and $a + d$. Find the roots and the value of k. **[6]**

4 You are given that if $\mathbf{M} = \begin{pmatrix} 4 & 0 & 1 \\ -6 & 1 & 1 \\ 5 & 2 & 5 \end{pmatrix}$ then $\mathbf{M}^{-1} = \dfrac{1}{k}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}$.

Find the value of k. Hence solve the following simultaneous equations. **[6]**

$$\begin{aligned} 4x \quad\quad + \quad z &= 9 \\ -6x + \quad y + \quad z &= 32 \\ 5x + 2y + 5z &= 81 \end{aligned}$$

5 Use standard series formulae to show that $\sum_{r=1}^{n}(r+2)(r-3) = \frac{1}{3}n(n^2 - 19)$. **[6]**

6 Prove by induction that $1 \times 2 + 2 \times 3 + \ldots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$ for all positive integers n. **[6]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{5x - 9}{(2x - 3)(2x + 7)}$.

(i) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

(ii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your answers. **[3]**

(iii) Sketch the curve. **[3]**

(iv) Solve the inequality $\dfrac{5x - 9}{(2x - 3)(2x + 7)} \leqslant 0$. **[3]**

8 **(a)** Fig. 8 shows an Argand diagram.

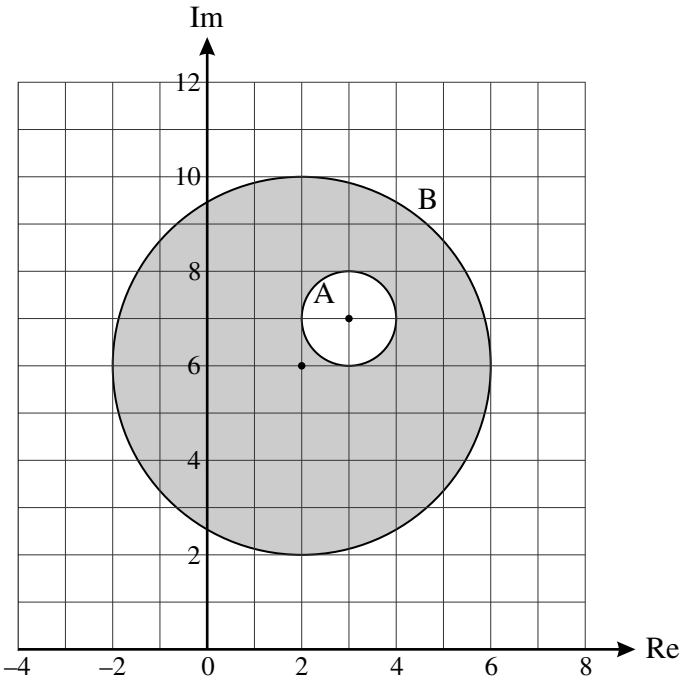

Fig. 8

(i) Write down the equation of the locus represented by the circumference of circle B. **[3]**

(ii) Write down the two inequalities that define the shaded region between, but not including, circles A and B. **[3]**

(b) **(i)** Draw an Argand diagram to show the region where

$$\frac{\pi}{4} < \arg\big(z - (2 + \mathrm{j})\big) < \frac{3\pi}{4}.$$ **[3]**

(ii) Determine whether the point $43 + 47\mathrm{j}$ lies within this region. **[3]**

9 **(i)** Verify that $\dfrac{4+r}{r(r+1)(r+2)} = \dfrac{2}{r} - \dfrac{3}{r+1} + \dfrac{1}{r+2}$. **[2]**

(ii) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{4+r}{r(r+1)(r+2)} = \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2}.$$ **[6]**

(iii) Write down the limit to which $\displaystyle\sum_{r=1}^{n} \frac{4+r}{r(r+1)(r+2)}$ converges as n tends to infinity. **[1]**

(iv) Find $\displaystyle\sum_{r=50}^{100} \frac{4+r}{r(r+1)(r+2)}$, giving your answer to 3 significant figures. **[3]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
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Section A (36 marks)

1 Find the values of A, B and C in the identity $4x^2 - 16x + C \equiv A(x + B)^2 + 2$. **[4]**

2 You are given that $\mathbf{M} = \begin{pmatrix} 2 & -5 \\ 3 & 7 \end{pmatrix}$.

$\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \end{pmatrix}$ represents two simultaneous equations.

(i) Write down these two equations. **[2]**

(ii) Find $\mathbf{M}^{-1}$ and use it to solve the equations. **[4]**

3 The cubic equation $2z^3 - z^2 + 4z + k = 0$, where k is real, has a root $z = 1 + 2\text{j}$.

Write down the other complex root. Hence find the real root and the value of k. **[6]**

4 The roots of the cubic equation $x^3 - 2x^2 - 8x + 11 = 0$ are α, β and γ.

Find the cubic equation with roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$. **[6]**

5 Use the result $\dfrac{1}{5r-1} - \dfrac{1}{5r+4} \equiv \dfrac{5}{(5r-1)(5r+4)}$ and the method of differences to find

$$\sum_{r=1}^{n} \frac{1}{(5r-1)(5r+4)},$$

simplifying your answer. **[6]**

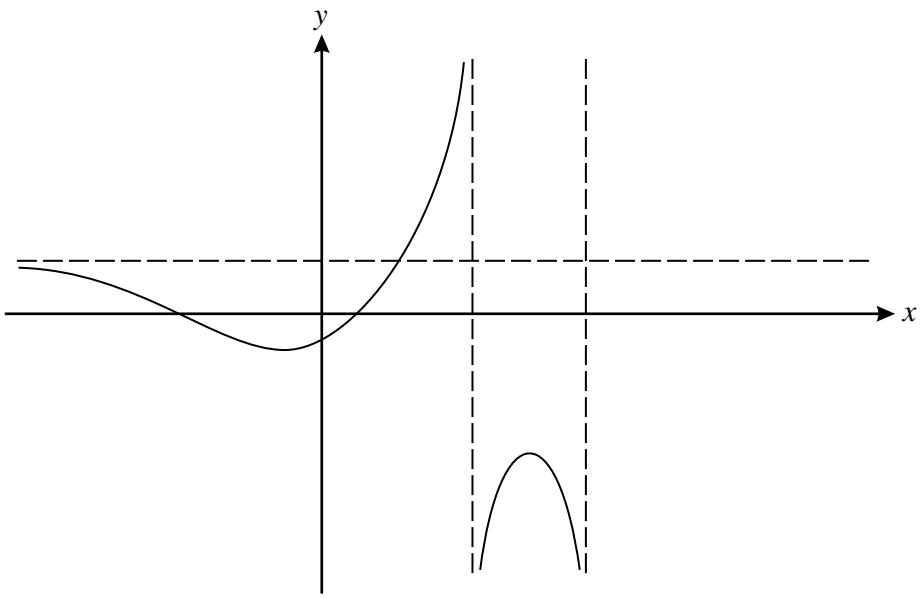
6 A sequence is defined by $u_1 = 2$ and $u_{n+1} = \dfrac{u_n}{1 + u_n}$.

(i) Calculate u_3. **[2]**

(ii) Prove by induction that $u_n = \dfrac{2}{2n-1}$. **[6]**

Section B (36 marks)

7 Fig. 7 shows an incomplete sketch of $y = \dfrac{(2x-1)(x+3)}{(x-3)(x-2)}$.



Not to scale

Fig. 7

(i) Find the coordinates of the points where the curve cuts the axes. **[2]**

(ii) Write down the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for large positive values of x, justifying your answer. Copy and complete the sketch. **[3]**

(iv) Solve the inequality $\dfrac{(2x-1)(x+3)}{(x-3)(x-2)} < 2$. **[4]**

8 Two complex numbers, α and β, are given by $\alpha = \sqrt{3} + \mathrm{j}$ and $\beta = 3\mathrm{j}$.

(i) Find the modulus and argument of α and β. **[3]**

(ii) Find $\alpha\beta$ and $\dfrac{\beta}{\alpha}$, giving your answers in the form $a + b\mathrm{j}$, showing your working. **[5]**

(iii) Plot α, β, $\alpha\beta$ and $\dfrac{\beta}{\alpha}$ on a single Argand diagram. **[2]**

[Question 9 is printed overleaf.]

9 The matrices $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ represent transformations P and Q respectively.

(i) Describe fully the transformations P and Q. **[4]**

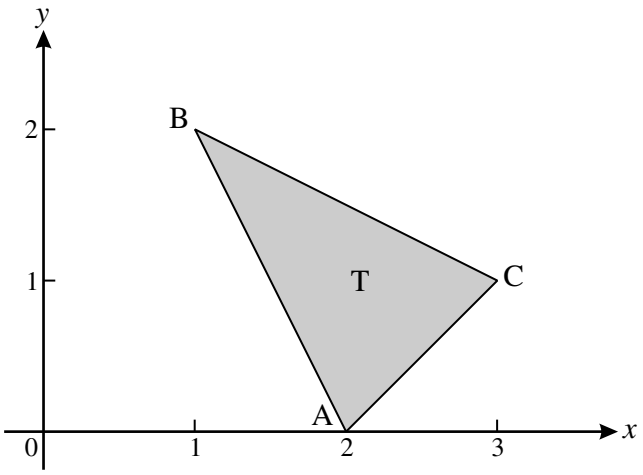


Fig. 9

Fig. 9 shows triangle T with vertices A (2, 0), B (1, 2) and C (3, 1).

Triangle T is transformed first by transformation P, then by transformation Q.

(ii) Find the single matrix that represents this composite transformation. **[2]**

(iii) This composite transformation maps triangle T onto triangle T′, with vertices A′, B′ and C′. Calculate the coordinates of A′, B′ and C′. **[2]**

T′ is reflected in the line $y = -x$ to give a new triangle, T″.

(iv) Find the matrix **R** that represents reflection in the line $y = -x$. **[2]**

(v) A single transformation maps T″ onto the original triangle, T. Find the matrix representing this transformation. **[4]**

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ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4755
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

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These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
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- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Find the values of P, Q, R and S in the identity $3x^3 + 18x^2 + Px + 31 \equiv Q(x + R)^3 + S$. **[5]**

2 You are given that $\mathbf{M} = \begin{pmatrix} 4 & 0 \\ -1 & 3 \end{pmatrix}$.

(i) The transformation associated with **M** is applied to a figure of area 3 square units. Find the area of the transformed figure. **[2]**

(ii) Find $\mathbf{M}^{-1}$ and $\det \mathbf{M}^{-1}$. **[3]**

(iii) Explain the significance of $\det \mathbf{M} \times \det \mathbf{M}^{-1}$ in terms of transformations. **[2]**

3 The roots of the cubic equation $x^3 - 4x^2 + 8x + 3 = 0$ are α, β and γ.

Find a cubic equation whose roots are $2\alpha - 1$, $2\beta - 1$ and $2\gamma - 1$. **[7]**

4 Represent on an Argand diagram the region defined by $2 < |z - (3 + 2\mathrm{j})| \leqslant 3$. **[6]**

5 Use standard series formulae to show that $\sum_{r=1}^{n} r^2(3 - 4r) = \frac{1}{2}n(n + 1)(1 - 2n^2)$. **[5]**

6 A sequence is defined by $u_1 = 5$ and $u_{n+1} = u_n + 2^{n+1}$. Prove by induction that $u_n = 2^{n+1} + 1$. **[6]**

Section B (36 marks)

7 Fig. 7 shows part of the curve with equation $y = \dfrac{x+5}{(2x-5)(3x+8)}$.

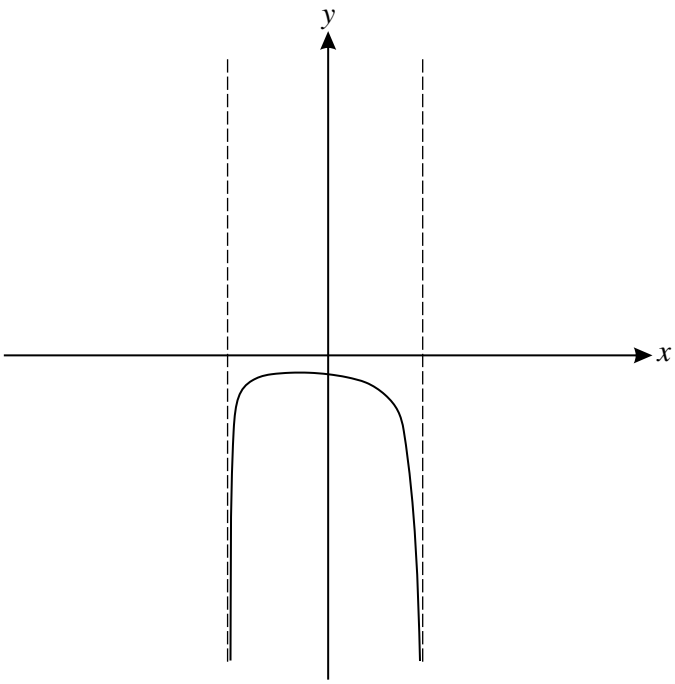

Fig. 7

(i) Write down the coordinates of the two points where the curve crosses the axes. **[2]**

(ii) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

(iii) Determine how the curve approaches the horizontal asymptote for large positive and large negative values of x. **[3]**

(iv) On the copy of Fig. 7, sketch the rest of the curve. **[2]**

(v) Solve the inequality $\dfrac{x+5}{(2x-5)(3x+8)} < 0$. **[2]**

8 The function $\mathrm{f}(z) = z^4 - z^3 + az^2 + bz + c$ has real coefficients. The equation $\mathrm{f}(z) = 0$ has roots α, β, γ and δ where $\alpha = 1$ and $\beta = 1 + \mathrm{j}$.

(i) Write down the other complex root and explain why the equation must have a second real root. **[2]**

(ii) Write down the value of $\alpha + \beta + \gamma + \delta$ and find the second real root. **[3]**

(iii) Find the values of a, b and c. **[5]**

(iv) Write down $\mathrm{f}(-z)$ and the roots of $\mathrm{f}(-z) = 0$. **[2]**

9 You are given that $\mathbf{A} = \begin{pmatrix} -2 & 1 & -5 \\ 3 & a & 1 \\ 1 & -1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2a+1 & 3 & 1+5a \\ -5 & 1 & -13 \\ -3-a & -1 & -2a-3 \end{pmatrix}$.

(i) Show that $\mathbf{AB} = (8+a)\mathbf{I}$. **[3]**

(ii) State the value of a for which $\mathbf{A}^{-1}$ does not exist. Write down $\mathbf{A}^{-1}$ in terms of a, when $\mathbf{A}^{-1}$ exists. **[3]**

(iii) Use $\mathbf{A}^{-1}$ to solve the following simultaneous equations. **[5]**

$$\begin{aligned} -2x + y - 5z &= -55 \\ 3x + 4y + z &= -9 \\ x - y + 2z &= 26 \end{aligned}$$

(iv) What can you say about the solutions of the following simultaneous equations? **[1]**

$$\begin{aligned} -2x + y - 5z &= p \\ 3x - 8y + z &= q \\ x - y + 2z &= r \end{aligned}$$

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ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4755
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Friday 20 May 2011
Afternoon

Duration: 1 hour 30 minutes

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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1 **(i)** Write down the matrix for a rotation of 90° anticlockwise about the origin. **[1]**

(ii) Write down the matrix for a reflection in the line $y = x$. **[1]**

(iii) Find the matrix for the composite transformation of rotation of 90° anticlockwise about the origin, followed by a reflection in the line $y = x$. **[2]**

(iv) What single transformation is equivalent to this composite transformation? **[1]**

2 You are given that $z = 3 - 2\text{j}$ and $w = -4 + \text{j}$.

(i) Express $\dfrac{z + w}{w}$ in the form $a + b\text{j}$. **[3]**

(ii) Express w in modulus-argument form. **[3]**

(iii) Show w on an Argand diagram, indicating its modulus and argument. **[2]**

3 The equation $x^3 + px^2 + qx + 3 = 0$ has roots α, β and γ, where

$$\alpha + \beta + \gamma = 4$$
$$\alpha^2 + \beta^2 + \gamma^2 = 6.$$

Find p and q. **[5]**

4 Solve the inequality $\dfrac{5x}{x^2 + 4} < x$. **[6]**

5 Given that $\dfrac{3}{(3r-1)(3r+2)} \equiv \dfrac{1}{3r-1} - \dfrac{1}{3r+2}$, find $\displaystyle\sum_{r=1}^{20} \frac{1}{(3r-1)(3r+2)}$, giving your answer as an exact fraction. **[5]**

6 Prove by induction that $1 + 8 + 27 + \ldots + n^3 = \frac{1}{4}n^2(n+1)^2$. **[7]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{(x+9)(3x-8)}{x^2-4}$.

(i) Write down the coordinates of the points where the curve crosses the axes. **[3]**

(ii) Write down the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(*A*) large positive values of x,

(*B*) large negative values of x. **[3]**

(iv) Sketch the curve. **[3]**

8 A polynomial P(z) has real coefficients. Two of the roots of P(z) = 0 are $2 - \mathrm{j}$ and $-1 + 2\mathrm{j}$.

(i) Explain why P(z) cannot be a cubic. **[1]**

You are given that P(z) is a quartic.

(ii) Write down the other roots of P(z) = 0 and hence find P(z) in the form $z^4 + az^3 + bz^2 + cz + d$. **[8]**

(iii) Show the roots of P(z) = 0 on an Argand diagram and give, in terms of z, the equation of the circle they lie on. **[2]**

9 The simultaneous equations

$$\begin{aligned} 2x - \ \ y &= 1 \\ 3x + ky &= b \end{aligned}$$

are represented by the matrix equation $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ b \end{pmatrix}$.

(i) Write down the matrix $\mathbf{M}$. **[2]**

(ii) State the value of k for which $\mathbf{M}^{-1}$ does not exist and find $\mathbf{M}^{-1}$ in terms of k when $\mathbf{M}^{-1}$ exists.

Use $\mathbf{M}^{-1}$ to solve the simultaneous equations when $k = 5$ and $b = 21$. **[7]**

(iii) What can you say about the solutions of the equations when $k = -\frac{3}{2}$? **[1]**

(iv) The two equations can be interpreted as representing two lines in the x-y plane. Describe the relationship between these two lines

(*A*) when $k = 5$ and $b = 21$,

(*B*) when $k = -\frac{3}{2}$ and $b = 1$,

(*C*) when $k = -\frac{3}{2}$ and $b = \frac{3}{2}$. **[3]**

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Friday 20 January 2012 – Afternoon

AS GCE MATHEMATICS (MEI)

4755 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

1 You are given that $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & p & -4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & q \\ 2 & -2 \\ 1 & -3 \end{pmatrix}$.

(i) Find **AB**. **[3]**

(ii) Hence prove that matrix multiplication is not commutative. **[2]**

2 Find the values of A, B, C and D in the identity $2x^3 - 3 \equiv (x + 3)(Ax^2 + Bx + C) + D$. **[5]**

3 Given that $z = 6$ is a root of the cubic equation $z^3 - 10z^2 + 37z + p = 0$, find the value of p and the other roots. **[6]**

4 Using the standard summation formulae, find $\sum_{r=1}^{n} r^2(r - 1)$. Give your answer in a fully factorised form. **[6]**

5 The equation $z^3 - 5z^2 + 3z - 4 = 0$ has roots α, β and γ. Find the cubic equation whose roots are $\frac{\alpha}{2} + 1$, $\frac{\beta}{2} + 1$, $\frac{\gamma}{2} + 1$, expressing your answer in a form with integer coefficients. **[6]**

6 Prove by induction that $\sum_{r=1}^{n} r3^{r-1} = \frac{1}{4}[3^n(2n - 1) + 1]$. **[8]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{(x + 1)(2x - 1)}{x^2 - 3}$.

(i) Find the coordinates of the points where the curve crosses the axes. **[2]**

(ii) Write down the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or from below for

(*A*) large positive values of x,

(*B*) large negative values of x. **[3]**

(iv) Sketch the curve. **[3]**

(v) Solve the inequality $\dfrac{(x + 1)(2x - 1)}{x^2 - 3} < 2$. **[3]**

8 **(i)** Sketch on an Argand diagram the locus, C, of points for which $|z - 4| = 3$. **[3]**

(ii) By drawing appropriate lines through the origin, indicate on your Argand diagram the point A on the locus C where $\arg z$ has its maximum value. Indicate also the point B on the locus C where $\arg z$ has its minimum value. **[2]**

(iii) Given that $\arg z = \alpha$ at A and $\arg z = \beta$ at B, indicate on your Argand diagram the set of points for which $\beta \leqslant \arg z \leqslant \alpha$ and $|z - 4| \geqslant 3$. **[2]**

(iv) Calculate the value of α and the value of β. **[3]**

9 The matrix $\mathbf{R}$ is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(i) Explain in terms of transformations why $\mathbf{R}^4 = \mathbf{I}$. **[3]**

(ii) Describe the transformation represented by $\mathbf{R}^{-1}$ and write down the matrix $\mathbf{R}^{-1}$. **[2]**

(iii) $\mathbf{S}$ is the matrix representing rotation through 60° anticlockwise about the origin. Find $\mathbf{S}$. **[2]**

(iv) Write down the smallest positive integers m and n such that $\mathbf{S}^m = \mathbf{R}^n$, explaining your answer in terms of transformations. **[2]**

(v) Find $\mathbf{RS}$ and explain in terms of transformations why $\mathbf{RS} = \mathbf{SR}$. **[3]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE

OCR
RECOGNISING ACHIEVEMENT

Friday 18 May 2012 – Morning

AS GCE MATHEMATICS (MEI)

4755 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

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INFORMATION FOR CANDIDATES

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INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 You are given that the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a transformation A, and that the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents a transformation B.

(i) Describe the transformations A and B. **[2]**

(ii) Find the matrix representing the composite transformation consisting of A followed by B. **[2]**

(iii) What single transformation is represented by this matrix? **[1]**

2 You are given that z_1 and z_2 are complex numbers.

$z_1 = 3 + 3\sqrt{3}\text{j}$, and z_2 has modulus 5 and argument $\frac{\pi}{3}$.

(i) Find the modulus and argument of z_1, giving your answers exactly. **[4]**

(ii) Express $z_2$in the form $a + b\text{j}$, where a and b are to be given exactly. **[2]**

(iii) Explain why, when plotted on an Argand diagram, z_1, z_2 and the origin lie on a straight line. **[1]**

3 The cubic equation $3x^3 + 8x^2 + px + q = 0$ has roots α, $\frac{\alpha}{6}$ and $\alpha - 7$. Find the values of α, p and q. **[6]**

4 Solve the inequality $\frac{3}{x-4} > 1$. **[4]**

5 **(i)** Show that $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$. **[2]**

(ii) Use the method of differences to find $\sum_{r=1}^{30} \frac{1}{(2r+1)(2r+3)}$, expressing your answer as a fraction. **[5]**

6 A sequence is defined by $a_1 = 1$ and $a_{k+1} = 3(a_k + 1)$.

(i) Calculate the value of the third term, a_3. **[1]**

(ii) Prove by induction that $a_n = \frac{5 \times 3^{n-1} - 3}{2}$. **[6]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{x^2 - 25}{(x - 3)(x + 4)(3x + 2)}$.

(i) Write down the coordinates of the points where the curve crosses the axes. **[3]**

(ii) Write down the equations of the asymptotes. **[4]**

(iii) Determine how the curve approaches the horizontal asymptote for large positive values of x, and for large negative values of x. **[3]**

(iv) Sketch the curve. **[4]**

8 **(i)** Verify that $1 + 3j$ is a root of the equation $3z^3 - 2z^2 + 22z + 40 = 0$, showing your working. **[4]**

(ii) Explain why the equation must have exactly one real root. **[1]**

(iii) Find the other roots of the equation. **[5]**

9 You are given that $\mathbf{A} = \begin{pmatrix} -3 & -4 & 1 \\ 2 & 1 & k \\ 7 & -1 & -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 & -5 & 11 \\ -19 & -4 & -7 \\ -9 & -31 & 2 - k \end{pmatrix}$ and

$\mathbf{AB} = \begin{pmatrix} 79 & 0 & -3 - k \\ -9k - 27 & -31k - 14 & q \\ p & 0 & 82 + k \end{pmatrix}$ where p and q are to be determined.

(i) Show that $p = 0$ and $q = 15 + 2k - k^2$. **[3]**

It is now given that $k = -3$.

(ii) Find $\mathbf{AB}$ and hence write down the inverse matrix $\mathbf{A}^{-1}$. **[5]**

(iii) Use a matrix method to find the values of x, y and z that satisfy the equation $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ -23 \\ 9 \end{pmatrix}$. **[4]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE

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RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
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- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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Section A (36 marks)

1 Transformation A is represented by matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and transformation B is represented by matrix $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.

(i) Describe transformations A and B. **[3]**

(ii) Find the matrix for the composite transformation A followed by B. **[2]**

2 Given that $z = a + b\mathrm{j}$, find $\mathrm{Re}\left(\frac{z}{z^*}\right)$ and $\mathrm{Im}\left(\frac{z}{z^*}\right)$. **[4]**

3 You are given that $z = 2 + \mathrm{j}$ is a root of the cubic equation $2z^3 + pz^2 + 22z - 15 = 0$, where p is real. Find the other roots and the value of p. **[6]**

4 **(i)** Show that $x^2 - x + 2 > 0$ for all real x. **[2]**

(ii) Solve the inequality $\frac{2x}{x^2 - x + 2} > x$. **[5]**

5 You are given that $\frac{3}{(5 + 3x)(2 + 3x)} \equiv \frac{1}{2 + 3x} - \frac{1}{5 + 3x}$.

(i) Use this result to find $\sum_{r=1}^{100} \frac{1}{(5 + 3r)(2 + 3r)}$, giving your answer as an exact fraction. **[5]**

(ii) Write down the limit to which $\sum_{r=1}^{n} \frac{1}{(5 + 3r)(2 + 3r)}$ converges as n tends to infinity. **[1]**

6 Prove by induction that $1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n + 1)}{2}$. **[8]**

Section B (36 marks)

7 Fig. 7 shows a sketch of $y = \dfrac{x-4}{(x-5)(x-8)}$.

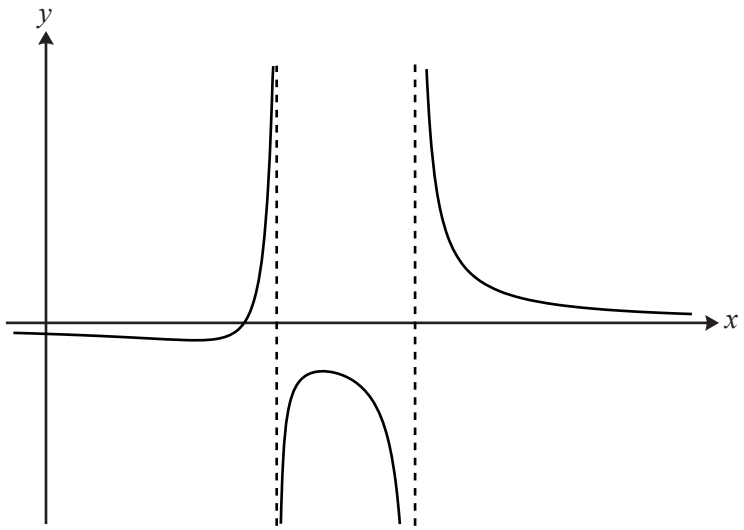

Fig. 7

(i) Write down the equations of the three asymptotes and the coordinates of the points where the curve crosses the axes. Hence write down the solution of the inequality $\dfrac{x-4}{(x-5)(x-8)} > 0$. **[6]**

(ii) The equation $\dfrac{x-4}{(x-5)(x-8)} = k$ has no real solutions. Show that $-1 < k < -\dfrac{1}{9}$. Relate this result to the graph of $y = \dfrac{x-4}{(x-5)(x-8)}$. **[7]**

8 **(i)** Indicate on an Argand diagram the set of points z for which $|z - (-8 + 15\mathrm{j})| < 10$. **[4]**

(ii) Using the diagram, show that $7 < |z| < 27$. **[3]**

(iii) Mark on your Argand diagram the point, P, at which $|z - (-8 + 15\mathrm{j})| = 10$ and $\arg z$ takes its maximum value. Find the modulus and argument of z at P. **[4]**

9 You are given that $\mathbf{A} = \begin{pmatrix} 8 & -7 & -12 \\ -10 & 5 & 15 \\ -9 & 6 & 6 \end{pmatrix}$ and $\mathbf{A}^{-1} = k\begin{pmatrix} 4 & 2 & 3 \\ 5 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix}$.

(i) Find the exact value of k. **[2]**

(ii) Using your answer to part **(i)**, solve the following simultaneous equations.

$$\begin{aligned} 8x - 7y - 12z &= 14 \\ -10x + 5y + 15z &= -25 \\ -9x + 6y + 6z &= 3 \end{aligned}$$ **[4]**

You are also given that $\mathbf{B} = \begin{pmatrix} -7 & 5 & 15 \\ a & -8 & -21 \\ 2 & -1 & -3 \end{pmatrix}$ and $\mathbf{B}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 0 & 5 \\ -4 & -3 & 1 \\ 2 & 1 & b \end{pmatrix}$.

(iii) Find the values of a and b. **[2]**

(iv) Write down an expression for $(\mathbf{AB})^{-1}$ in terms of $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$. Hence find $(\mathbf{AB})^{-1}$. **[4]**

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RECOGNISING ACHIEVEMENT

Monday 13 May 2013 – Afternoon

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Find the values of A, B, C and D in the identity $2x(x^2 - 5) \equiv (x - 2)(Ax^2 + Bx + C) + D$. **[5]**

2 You are given that $z = \frac{3}{2}$ is a root of the cubic equation $2z^3 + 9z^2 + 2z - 30 = 0$. Find the other two roots. **[6]**

3 You are given that $\mathbf{N} = \begin{pmatrix} -9 & -2 & -4 \\ 3 & 2 & 2 \\ 5 & 1 & 2 \end{pmatrix}$ and $\mathbf{N}^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ -\frac{7}{2} & p & -6 \end{pmatrix}$.

(i) Find the value of p. **[2]**

(ii) Solve the equation $\mathbf{N}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$. **[4]**

4 The complex number z_1 is $3 - 2\text{j}$ and the complex number z_2 has modulus 5 and argument $\frac{\pi}{4}$.

(i) Express z_2 in the form $a + b\text{j}$, giving a and b in exact form. **[2]**

(ii) Represent z_1, z_2, $z_1 + z_2$ and $z_1 - z_2$ on a single Argand diagram. **[4]**

5 You are given that $\frac{4}{(4n-3)(4n+1)} \equiv \frac{1}{4n-3} - \frac{1}{4n+1}$. Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{n}{4n+1}.$$ **[6]**

6 The cubic equation $x^3 - 5x^2 + 3x - 6 = 0$ has roots α, β and γ. Find a cubic equation with roots $\frac{\alpha}{3} + 1$, $\frac{\beta}{3} + 1$ and $\frac{\gamma}{3} + 1$, simplifying your answer as far as possible. **[7]**

Section B (36 marks)

7 Fig. 7 shows an incomplete sketch of $y = \dfrac{cx^2}{(bx-1)(x+a)}$ where a, b and c are integers. The asymptotes of the curve are also shown.

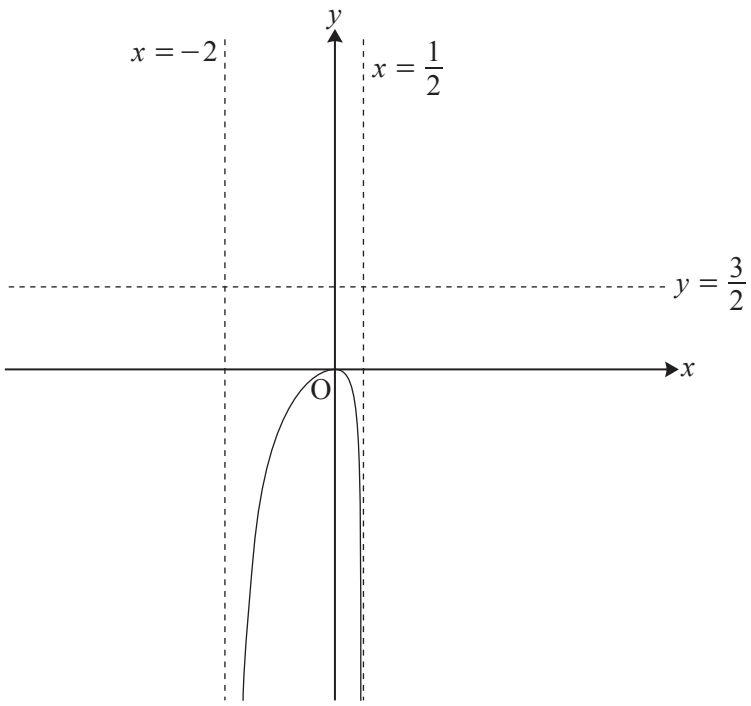

Fig. 7

(i) Determine the values of a, b and c. **[4]**

Use these values of a, b and c throughout the rest of the question.

(ii) Determine how the curve approaches the horizontal asymptote for large positive values of x, and for large negative values of x, justifying your answer. On the copy of Fig. 7, sketch the rest of the curve. **[4]**

(iii) Find the x coordinates of the points on the curve where $y = 1$. Write down the solution to the inequality $\dfrac{cx^2}{(bx-1)(x+a)} < 1$. **[4]**

8 **(i)** Use standard series formulae to show that

$$\sum_{r=1}^{n}[r(r-1)-1] = \frac{1}{3}n(n+2)(n-2). \qquad (*)$$ **[5]**

(ii) Prove (*) by mathematical induction. **[7]**

9 **(i)** Describe fully the transformation Q, represented by the matrix $\mathbf{Q}$, where $\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. **[2]**

The transformation M is represented by the matrix $\mathbf{M}$, where $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$.

(ii) M maps all points on the line $y = 2$ onto a single point, P. Find the coordinates of P. **[2]**

(iii) M maps all points on the plane onto a single line, l. Find the equation of l. **[2]**

(iv) M maps all points on the line n onto the point $(-6,\ 6)$. Find the equation of n. **[2]**

(v) Show that $\mathbf{M}$ is singular. Relate this to the transformation it represents. **[2]**

(vi) R is the composite transformation M followed by Q. R maps all points on the plane onto the line q. Find the equation of q. **[2]**

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Friday 16 May 2014 – Afternoon

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

1 Use standard series formulae to find $\sum_{r=1}^{n} r(r-2)$, factorising your answer as far as possible. **[5]**

2 Fig. 2 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

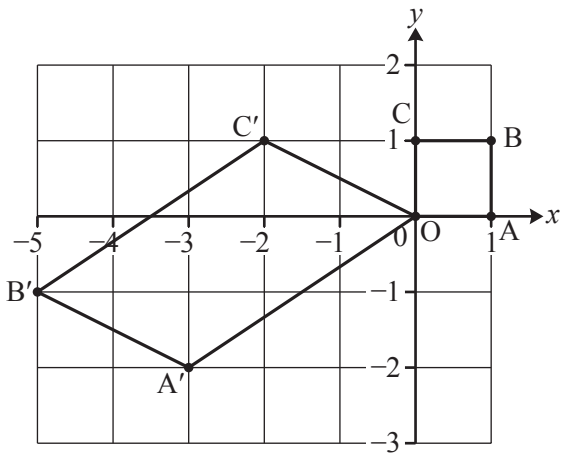

Fig. 2

(i) Write down the matrix **T** representing this transformation. **[2]**

The quadrilateral OA′B′C′ is reflected in the x-axis to give a new quadrilateral, OA″B″C″.

(ii) Write down the matrix representing reflection in the x-axis. **[1]**

(iii) Find the single matrix that will transform OABC onto OA″B″C″. **[2]**

3 You are given that $z = 2+3\mathrm{j}$ is a root of the quartic equation $z^4 - 5z^3 + 15z^2 - 5z - 26 = 0$. Find the other roots. **[7]**

4 Use the identity $\frac{1}{2r+3} - \frac{1}{2r+5} \equiv \frac{2}{(2r+3)(2r+5)}$ and the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2r+3)(2r+5)}$, expressing your answer as a single fraction. **[5]**

5 The roots of the cubic equation $3x^3 - 9x^2 + x - 1 = 0$ are α, β and γ. Find the cubic equation whose roots are $3\alpha - 1$, $3\beta - 1$ and $3\gamma - 1$, expressing your answer in a form with integer coefficients. **[7]**

6 Prove by induction that $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$. **[7]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{x^2 - 5}{(x+3)(x-2)(ax-1)}$, where a is a constant.

(i) Find the coordinates of the points where the curve crosses the x-axis and the y-axis. **[2]**

(ii) You are given that the curve has a vertical asymptote at $x = \frac{1}{2}$. Write down the value of a and the equations of the other asymptotes. **[3]**

(iii) Sketch the curve. **[4]**

(iv) Find the set of values of x for which $y > 0$. **[3]**

8 You are given the complex number $w = 2 + 2\sqrt{3}\,\mathrm{j}$.

(i) Express w in modulus-argument form. **[3]**

(ii) Indicate on an Argand diagram the set of points, z, which satisfy both of the following inequalities.

$$-\frac{\pi}{2} \leqslant \arg z \leqslant \frac{\pi}{3} \text{ and } |z| \leqslant 4$$

Mark w on your Argand diagram and find the greatest value of $|z - w|$. **[9]**

9 You are given that $\mathbf{A} = \begin{pmatrix} 1 & 3 & -1 \\ -1 & \alpha & -1 \\ -2 & -1 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3\alpha - 1 & -8 & \alpha - 3 \\ 5 & 1 & 2 \\ 2\alpha + 1 & -5 & \alpha + 3 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} \gamma & 0 & 0 \\ \beta & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix}$.

(i) Show that $\beta = 0$. **[2]**

(ii) Find γ in terms of α. **[2]**

(iii) Write down $\mathbf{A}^{-1}$ for the case when $\alpha = 2$. State the value of α for which $\mathbf{A}^{-1}$ does not exist. **[3]**

(iv) Use your answer to part **(iii)** to solve the following simultaneous equations.

$$\begin{aligned} x + 3y - z &= 25 \\ -x + 2y - z &= 11 \\ -2x - y + 3z &= -23 \end{aligned}$$ **[5]**

END OF QUESTION PAPER